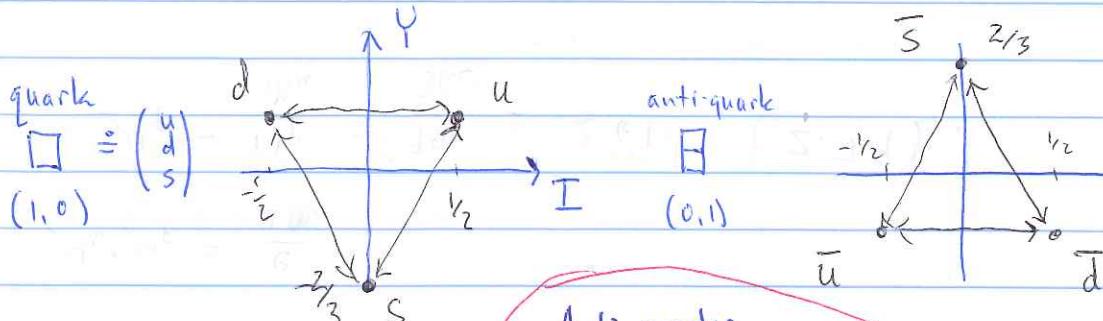


Thurs. 6 February 2014

Flavor SU(3) symmetry



Anti-quarks
have (-) QM of quarks

$$\begin{aligned} Y &= \text{hypercharge} \\ &= B + S \\ &\quad \uparrow \quad \text{strangeness} \\ &\quad \text{baryon} \\ &\quad \text{number} \end{aligned}$$

$$Y = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} T_8 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

$$Q = I_3 + \frac{1}{2} Y$$

$$\begin{aligned} Q|u\rangle &= \left(\frac{1}{2} + \frac{1}{2} - \frac{1}{3}\right)|u\rangle & Q|d\rangle &= \left(-\frac{1}{2} + \frac{1}{2} - \frac{1}{3}\right)|d\rangle \\ &= \frac{2}{3}|u\rangle & &= -\frac{1}{3}|d\rangle \end{aligned}$$

$$\begin{aligned} Q|s\rangle &= \left(0 + \frac{1}{2} - \frac{2}{3}\right)|s\rangle \\ &= -\frac{1}{3}|s\rangle \end{aligned}$$

NOTE: $\hat{S}|s\rangle = -1|s\rangle$ strange quark has -1 strangeness
 $\hat{S}|\bar{s}\rangle = +1|\bar{s}\rangle$ anti- +1

B associated with U(1) symmetry

I, S associated with SU(3) symmetry

I, B, S (Y) all conserved quantum #'s of strong interactions

consequently Q is conserved by strong interactions

Flavor SU(3) is not an exact theory of nature, as we will see. This is an example of how using perturbation theory to 1) construct a symmetric theory 2) perturbatively add explicit symmetry breaking operators

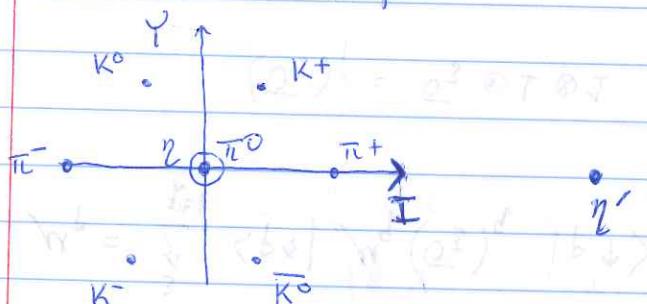
Construct Meson states as combinations of anti-quark \otimes quark

$$\square \otimes \square = \square + \square$$

3 antiquarks

$$1 \oplus 8$$

flavor flavor 8
singlet octet



How does this look in terms of quarks?

Recall anti-quarks are representations of conjugate algebra

$$[t_a, t_b] = if_{abc} t_c$$

$$[t_a^*, t_b^*] = -if_{abc} t_c^*$$

$$\bar{t}_a = -t_a^*$$

$$[-t_a^*, -t_b^*] = if_{abc} (-t_c^*)$$

$$e^{i\theta_{ata}} \uparrow$$

$$e^{-i\theta_{ata}^*} \downarrow$$

Not right for modern nomenclature

(66)

What are conjugate raising & lowering operators?

$$t_+ = \frac{t_1 + it_2}{\sqrt{2}}$$

$$t_+^* = \frac{t_1^* - it_2^*}{\sqrt{2}} = \frac{\bar{t}_1 - i\bar{t}_2}{\sqrt{2}} = -\bar{t}_-$$

In general

$$\bar{E}_{\pm} = -E_{\mp}^*$$

Look at I_- on $|u^+\rangle$

$$|u^+\rangle = |u\bar{d}\rangle$$

so just apply matrix and do component analysis

$$\begin{aligned} I_- |u\bar{d}\rangle &= |(I-u)\bar{d}\rangle + |u(I^*\bar{d})\rangle \\ &= \frac{1}{\sqrt{2}} |\bar{d}\bar{d}\bar{d}\rangle + |u(-\bar{I}_+\bar{d})\rangle \\ &= \frac{1}{\sqrt{2}} |\bar{d}\bar{d}\rangle - \frac{1}{\sqrt{2}} |u\bar{u}\rangle = -|\pi^0\rangle \end{aligned}$$

$$\text{Recall } I_- = t_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and } |u\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |\bar{d}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |s\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

With our conjugation, we have the basis

$$|\bar{u}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |\bar{d}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |\bar{s}\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{I}_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Notes: no energy relation \rightarrow no conservation

(24)

(67)

What if we start with $|K^+\rangle$?

$$|u\bar{s}\rangle$$

lets apply V_-

$$V_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$V_- |u\bar{s}\rangle = |(V-u)\bar{s}\rangle + |u(V^*\bar{s})\rangle$$

$$= \frac{1}{\sqrt{2}} |s\bar{s}\rangle - \frac{1}{\sqrt{2}} |u\bar{u}\rangle$$

$$= \frac{1}{\sqrt{2}} \left(|s\bar{s}\rangle - \frac{1}{2} |u\bar{u}\rangle - \frac{1}{2} |d\bar{d}\rangle - \frac{1}{2} |u\bar{u}\rangle + \frac{1}{2} |d\bar{d}\rangle \right)$$

$$= -\frac{1}{2\sqrt{2}} (|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle) - \frac{1}{2} (|u\bar{u}\rangle - |d\bar{d}\rangle)$$

eigenstate of t_8
 $|q\rangle$

eigenstate of t_3
 $|\pi^0\rangle$

This makes sense. Isospin t_8 is a conserved QN & also $\gamma(t_8)$ and V_\pm mix change both I and γ . (diagonal)

$$[V_+, V_-] = \frac{\sqrt{3}}{2} H_2 + \frac{1}{2} H_1$$

\leftarrow roots (weights of adjoint)

$$(V_-)^2 |u\bar{s}\rangle$$

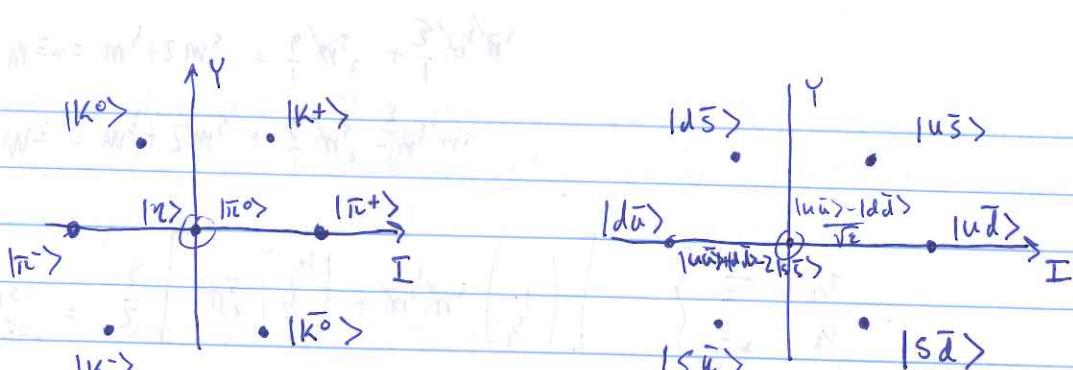
$$= \frac{1}{\sqrt{2}} V_- (|s\bar{s}\rangle - |u\bar{u}\rangle)$$

$$= \frac{1}{\sqrt{2}} \left(|(V-s)\bar{s}\rangle + |s(V^*\bar{s})\rangle - |(V-u)\bar{u}\rangle - |u(V^*\bar{u})\rangle \right)$$

$$= \frac{1}{\sqrt{2}} \left(\phi + \phi - \frac{1}{\sqrt{2}} |s\bar{u}\rangle - \frac{1}{\sqrt{2}} |s\bar{u}\rangle - \phi \right)$$

$$= -|s\bar{u}\rangle$$

$$\propto |K^-\rangle$$



$$J^{PC} = 0^{-+} \text{ pseudo-scalar mesons}$$

What about singlet? $3 \otimes \bar{3} = 8 \oplus 1$

$$|\eta'\rangle = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle$$

If flavor $SU(3)$ were an exact symmetry, then

the masses would all be degenerate.

$$m_{\pi^\pm} \approx 139.6 \text{ MeV} \quad m_{K^\pm} \approx 493.7 \text{ MeV}$$

$$m_{\pi^0} \approx 135.0 \text{ MeV} \quad m_{K^0} \approx 497.6 \text{ MeV}$$

$$m_\eta \approx 547.9 \text{ MeV}$$

$$m_{\eta'} \approx 957.8 \text{ MeV}$$

These masses don't seem similar...

$$\frac{m_{\eta'}}{m_{\pi^0}} \approx 7$$

As we will come to later, the flavored (8) 0^{-+} mesons are special, their masses are protected by an approximate flavor symmetry - $SU(3)$, while the η' , in fact has an "anomalous" $U(1)$ symmetry, and so its mass is not protected.

$$\begin{aligned} \text{tr}(t_3 \phi) &= \pi^0 \\ \text{tr}(t-\phi) &= \pi^+ \end{aligned}$$

The isospin splittings are much smaller than the strange splitting.

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{6}}\eta' \end{pmatrix}$$

Note: each generator $t_3, t_8, I^\pm, V^\pm, U^\pm$ picks out one state

(69)

$$(\bar{s}^2 \bar{s}^2) = \frac{1}{3} (\bar{s}^2 + \bar{s}^2 + \bar{s}^2) = \frac{1}{3} [6 \text{ gluons} - 3 \text{ gluons}]$$

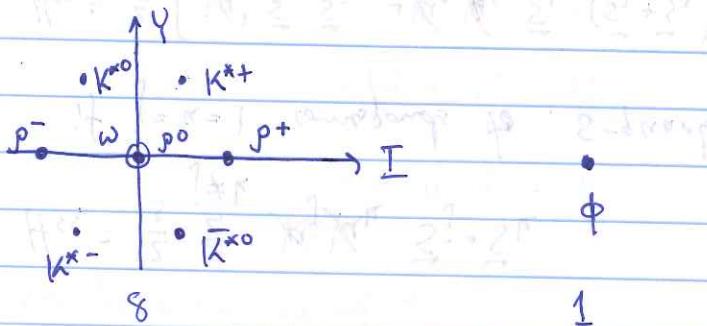
Before discussing the sense in which this $SU(3)$ symmetry is quantitatively useful, let's again assume the symmetry, and look at other hadrons.

0^+ mesons $|\bar{q} \gamma_5 q\rangle$

there are also important vector mesons

1^- $|\bar{q} \gamma_\mu q\rangle$ show up as strong resonances $\xrightarrow[T \text{ QCD}]{} \dots$

These will have the exact same flavor structure



$$M_{p^0} \approx 775.3 \text{ MeV}$$

$$M_{K^{*0}} \approx 891.7 \text{ MeV}$$

$$M_{p^+} \approx 775.1 \text{ MeV}$$

$$M_{K^{*+}} \approx 895.8 \text{ MeV}$$

$$\Rightarrow M_w \approx 782.7 \text{ MeV}$$

$$M_\phi \approx 1019.5 \text{ MeV}$$

Notice $M_w \approx M_{p^+}$

As we will learn later, there is strong mixing between the states $|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle \nparallel |u\bar{u} + d\bar{d} + s\bar{s}\rangle$

s.t. $|\phi\rangle \sim |s\bar{s}\rangle$ and $|w\rangle \sim |u\bar{u} + d\bar{d}\rangle$

What if one expands one of ϕ or w to expand

(84)

(70)

How about baryons?

Postulate that baryons composed of 3 quarks

$$3 \otimes 3 \otimes 3$$

$$3 \otimes 3 = 1 \otimes 1 = \boxed{1} \oplus \boxed{8}$$

$$6 \quad \bar{3}$$

$$3 \otimes 3 \otimes 3 = 1 \otimes (\boxed{1} \oplus \boxed{8})$$

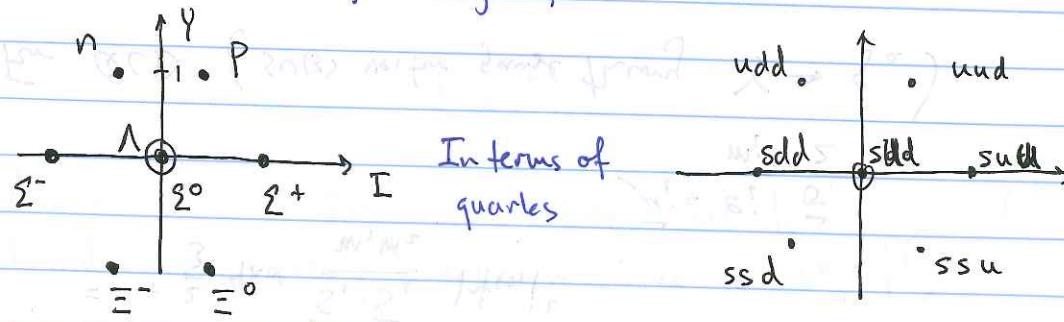
$$1 = \boxed{1} \oplus \boxed{8} + \boxed{8} + \boxed{1}$$

$$10 + 8 + 8 + 1 = 27$$

Let's focus on one of the 8's first $\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}$

This is symmetric and anti-symmetric or mixed symmetric

In our Y-I weight diagram, we have



$$M_{n,p} \sim 938.9 \text{ MeV}$$

$$M_\Lambda \sim 1115.7 \text{ MeV}$$

$$M_\Sigma \sim 1193.2 \text{ MeV}$$

$$M_\Xi \sim 1318.3 \text{ MeV}$$

(83)

(71)

Just as w/ mesons, we can write baryons in matrix form

$$B^{ij} = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & P \\ \Xi^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & N \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix} = \frac{1}{a} \frac{\text{Beta}_a}{\sqrt{6}}$$

The upper Σ , lower indices are to remind us the one index transforms like a 3 , and the other like a 3^*

$$\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} \xrightarrow[3^*]{3}$$

This is important, because as we saw with the mesons, this means there are important (-) signs. If the upper index denotes the 3^* and the lower the 3

$$[T_a]_j^i = \frac{1}{2} [\lambda_a]_{ij}$$

$$|T_a l_i\rangle = |l_j\rangle [T_a]_j^i$$

$$|T_a l^i\rangle = -|l^j\rangle [T_a]_j^i$$

A bra $\langle l_i |$ is the conjugate of a ket $|l_i\rangle$, so it should (and it does) transform like a 3^* $|l^i\rangle$

For mesons, this is clear. Think of $\Phi = \bar{q} q$.

(72)

Gell-Mann's proposal -

most of the baryon masses from "strong" interactions

splitting (ignoring Isospin) arise from an operator proportional to Strangeness (t_8).

$$\langle B | H_s | B \rangle = \langle B | H_0 | B \rangle + \langle B | \epsilon H_8 | B \rangle$$

flavor singlet flavor 8

To see how this works, we need to define d-constants

$$t_{ab} = \frac{1}{2} [t_a, t_b] + \frac{1}{2} \{t_a, t_b\}$$

$$= \frac{1}{2} i f_{abc} t_c + \frac{1}{2} \cdot \frac{1}{3} \delta_{ab} + \frac{1}{2} d_{abc} t_c$$

completely
anti-symmetric

completely
symmetric

These d-constants are 0 for
 $SU(2)$.

$\langle B | H_0 | B \rangle$ will give symmetric mass to all states

$\langle B | \epsilon H_8 | B \rangle$ will split masses

We wrote $B = ()$

$$\text{so } \langle B | \epsilon H_8 | B \rangle \propto \epsilon \text{ Tr}(\bar{B} T_8 B), \quad \bar{B} \propto \sum_a B_{aa}$$

$$\text{Tr}(T_a T_8 T_b) = ?$$

$$T_8 T_b = \frac{1}{2} [T_8, T_b] + \frac{1}{2} \{T_8, T_b\}$$

$$= \frac{1}{2} i f_{8bc} T_c + \frac{1}{2} \cdot \frac{1}{3} S_{8b} + \frac{1}{2} d_{8bc} T_c$$

$$\text{Tr}(T_a T_8 T_b) = \frac{1}{2} i f_{8bc} \langle T_a T_c \rangle + \frac{1}{2} \cdot \frac{1}{3} S_{8b} \langle T_a \rangle + \frac{1}{2} d_{8bc} \langle T_a T_c \rangle$$

$$= \frac{1}{4} f_{8ba} + \frac{1}{4} d_{8ba}$$

So instead, we can take

$$\text{Tr}(T_a \{T_8, T_b\}) = \frac{1}{2} d_{8ba}$$

$$\text{Tr}(T_a [T_8, T_b]) = \frac{1}{2} f_{8ba}$$

So we can write the L (H)

$$L = X \text{Tr}(\bar{B} T_8 B)$$

$$= \cancel{\alpha_1 \text{Tr}(\bar{B} B T_8)} + \cancel{\alpha_2}$$

$$= \varepsilon_1 \text{Tr}(\bar{B} T_8 B) + \varepsilon_2 \text{Tr}(\bar{B} B T_8)$$

$$= b_D \text{Tr}(\bar{B} \{T_8, B\}) + b_F \text{Tr}(\bar{B} [T_8, B])$$

$$b_D = \frac{1}{2} \varepsilon_1 + \frac{1}{2} \varepsilon_2$$

$$b_F = \frac{1}{2} \varepsilon_1 - \frac{1}{2} \varepsilon_2$$

$$\langle B | \varepsilon H_8 | B \rangle = \langle B | \varepsilon T_8 | B \rangle$$

$$= \varepsilon_1 \text{Tr}(B^+ T_8 B) + \varepsilon_2 \text{Tr}(B^+ B T_8)$$

$$T_8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 1 & -2 \end{pmatrix}$$

$$= \frac{\varepsilon_1}{2\sqrt{3}} \left([BB^+]_1^1 + [BB^+]_2^2 - 2[B B^+]_3^3 \right)$$

$$+ \frac{\varepsilon_2}{2\sqrt{3}} \left([B^+ B]_1^1 + [B^+ B]_2^2 - 2[B^+ B^+]_3^3 \right)$$

$$[B^+ B]_1^1 = \left| \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \right|^2 + |\Sigma^-|^2 + |\Xi^-|^2$$

$$[B^+ B]_2^2 = |\Sigma^+|^2 + \left| -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \right|^2 + |\Xi^0|^2$$

$$[B^+ B]_3^3 = |\bar{p}|^2 + |\bar{n}|^2 + \left| -2\frac{\Lambda}{\sqrt{6}} \right|^2$$

$$[B B^+]_1^1 = \left| \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \right|^2 + |\Sigma^+|^2 + |\bar{p}|^2$$

$$[B B^+]_2^2 = |\Sigma^-|^2 + \left| -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \right|^2 + |\bar{n}|^2$$

$$[B B^+]_3^3 = |\Xi^-|^2 + |\Xi^0|^2 + \left| -2\frac{\Lambda}{\sqrt{6}} \right|^2$$

$$\Rightarrow M_N = M_0 + \frac{\varepsilon_1}{2\sqrt{3}} - \frac{2\varepsilon_2}{2\sqrt{3}}$$

4 Masses

3 unknowns

$$M_\Lambda = M_0 - \frac{\varepsilon_1}{2\sqrt{3}} - \frac{\varepsilon_2}{2\sqrt{3}}$$

$$M_\Sigma = M_0 + \frac{\varepsilon_1}{2\sqrt{3}} + \frac{\varepsilon_2}{2\sqrt{3}}$$

1 relation

$$M_{\bar{\Xi}} = M_0 - 2\frac{\varepsilon_1}{2\sqrt{3}} + \frac{\varepsilon_2}{2\sqrt{3}}$$